Comment on "Critical Behavior of the Randomly Spin Diluted 2D Ising Model: A Grand Ensemble Approach"

In this Comment we want to point out that the grand ensemble approach applied in [1] suffers from being ill defined on the model under consideration.

We remind the reader that the grand ensemble approach which apparently goes back to Morita [2] consists in rewriting the weights in the quenched average

$$P(\underline{n}, \sigma) = P(\underline{n}) \frac{1}{Z_{\underline{n}}} \exp -H_{\underline{n}}(\sigma) \qquad (1)$$

as Gibbsian weights $\exp -H^{\phi}(\underline{n}, \sigma)/Z^{\phi}$ in an annealed average for some effective Hamiltonian $H^{\phi}(\underline{n}, \sigma)$. Here $\underline{n}$ denotes the occupation number variables, which are 0 or 1 independently on each site with a certain prescribed dilution probability, and σ denotes Ising spins, which are present on occupied sites, and interact at inverse temperature β via a nearest-neighbor interaction.

The "disorder potential" ϕ describes the difference between the original Hamiltonian and this effective Hamiltonian.

The assumption that an effective Hamiltonian exists for some given distribution (measure) is not an innocent one, as has been known for some time [3] . Indeed, recently [4,5] it was proved that in the thermodynamic limit there does *not* exist a well-behaved interaction potential, describing such an effective disorder-potential Hamiltonian. In other words, due to severe nonlocalities, these quenched measures are non-Gibbsian, for the model of Kühn [4], as well as for more general disordered models [5], just as in [3] various renormalized measures were shown to be.

We emphasize that this result occurs at low temperatures, but at arbitrary dilution. Hence critical points, as well as open regions in the dilution density-temperature plane around them, of the model studied in [1] are certainly affected. Thus the approximations used in [1] are intrinsically uncontrolled.

How reliable the conclusions reached in [1] are, remains therefore to be seen.

On the negative side, nonlocalities can of course strongly influence long-range properties, and critical properties are preeminently long-range properties.

On a more positive side, as in various other examples [6], one can show very generally [7] that these quenched measures belong to the "weakly Gibbsian" class, cf. [6,8–10]. Moreover, for the ferromagnetic Gibbs state, there is really an expansion of the (almost surely defined) interaction potential in terms of the form $\lambda_P \prod_{i \in P} n_i$, where P is running over the connected plaquettes on the lattice (as was used in [1]). Such an expansion does not always exist; for the random Dobrushin-state, for example, it does not (although an expansion of a different form does exist) [7].

It might be that this Gibbsian restoration of non-Gibbsian states (as carried out explicitly in, e.g., [10]) can to some extent explain that, as with renormalization group computations, often the results obtained by *a priori* mathematically objectionable methods turn out to be surprisingly good.

We claim that our results go some way in meeting the desire expressed in [1] that "a deeper understanding of our approach would ... be welcome."

Aernout C. D. van Enter*
Institute for Theoretical Physics
Rijksuniversiteit Groningen
Nijenborgh 4
Groningen, The Netherlands

C. Külske†
WIAS
Mohrenstrasse 39
Berlin, Germany

C. Maes‡
Instituut voor Theoretische Fysica
K. U. Leuven
Celestijnenlaan 200D
Leuven, Belgium

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*Electronic address: aenter@phys.rug.nl
†Electronic address: kuelske@wias-berlin.de
‡Electronic address: Christian.Maes@fys.kuleuven.be

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